The Cost and Duration of Cash-Balance Pension Plans

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Motivation

- Cash-balance plans are prevalent among employers across many industries. As of 2018, they make up 37% of all defined-benefit plans.

- Cash-balance plans have large asset size. In 2018, plan sponsors added $38.2B in contributions, increasing the total assets nationwide to $1.03T.

- This paper seeks to shed light on different cash-balance plans by looking at present-value cost and effective duration.

Objectives

- To calculate the present-value cost of funding cash-balance plan and the driving factors

- To derive effective duration of cash-balance liability

- To compare the costs of different cash-balance plans based on various IRS-sanctioned crediting alternatives

Background – Cash Balance Plan

- A cash-balance pension plan is a pension plan in which an employer credits a participant's account with a set percentage of his or her yearly compensation plus interest charges.

- Cash-balance plan is a defined-benefit plan whose funding limits, requirements and investment risks are based on defined-benefit requirements.

Background – Cash Balance Plan

- For example, assume a cash-balance plan where
  - Employer contribution is 5% of pay
  - Annual interest credit of 3%

- Now Joe joins this cash-balance plan as a new participant with a constant annual salary of $50,000.

		Account balance
Beginning of Year 1 account balance		$0
	Year 1 interest credit(3%*$0)	$0
	Year 1 contribution(5%*$50,000)	$2,500
End of Year 1 account balance		$2,500
	Year 2 interest credit(3%*$2,500)	$75
	Year 2 contribution(5%*$50,000)	$2,500
End of Year 2 account balance		$5,075

Background – IRS Guideline

- IRS maintains a suggested guidelines for crediting rates.

- If a company sets up its crediting rates in compliance with the guideline, they are exempted from the default rules for minimum lump-sum contribution rule.

Standard Index	Associated Margin
The discount rate on 3-month Treasury Bills	175 basis points
The discount rate on 6-month Treasury Bills or 12-month Treasury Bills	150 basis points
The yield on 1-year Treasury Constant Maturities	100 basis points
The yield on 2-year Treasury Constant Maturities or 3-year Treasury Constant Maturities	50 basis points
The yield on 5-year Treasury Constant Maturities or 7-year Treasury Constant Maturities	25 basis points
The yield on 10-year Treasury Constant Maturities or any longer period Treasury Constant Maturities	0 basis points
Annual rate of change of the Consumer Price Index	3 percentage points

Numerical Example

- Assume two hypothetical cash-balance liabilities:
  - Liability A credits at the two-year zero-coupon bond rate;
  - Liability B credits at the three-year zero-coupon bond rate;
  - Both plans have same starting balance and same time-to-exit of four years;
  - Future liabilities are discounted based on forward rates implicit in today's Treasury STRIP curve.

- Liability A's crediting rate $r_t^C \approx (f_t + f_{t+1})/2$. Its terminal cash balance is $B_4^A = B_0(1 + \frac{f_1+f_2}{2} + \frac{f_2+f_3}{2} + \frac{f_3+f_4}{2} + \frac{f_4+f_5}{2})$

- Liability B's crediting rate $r_t^C \approx (f_t + f_{t+1} + f_{t+2})/3$. Its terminal cash balance is $B_4^B = B_0(1 + \frac{f_1+f_2+f_3}{3} + \frac{f_2+f_3+f_4}{3} + \frac{f_3+f_4+f_5}{3} + \frac{f_4+f_5+f_6}{3})$

Numerical Example

- The discount rate for $t = 4$ is approx. $(1 + f_1 + f_2 + f_3 + f_4)$

- Liability A's present-value cost:

$$C_0^A = B_0 \frac{(1+\frac{f_1+f_2}{2}+\frac{f_2+f_3}{2}+\frac{f_3+f_4}{2}+\frac{f_4+f_5}{2})}{(1+f_1+f_2+f_3+f_4)} \approx B_0[1 + \frac{1}{2}(f_5 - f_1)]$$

- Liability B's present-value cost:

$$C_0^B = B_0 \frac{(1+\frac{f_1+f_2+f_3}{3}+\frac{f_2+f_3+f_4}{3}+\frac{f_3+f_4+f_5}{3}+\frac{f_4+f_5+f_6}{3})}{(1+f_1+f_2+f_3+f_4)} \approx B_0[1 + \frac{2}{3}(f_5 - f_2) + \frac{1}{3}(f_6 - f_1)]$$

Numerical Example - Cost

- Except for special cases, neither C_0^A nor C_0^B equals B_0

- Two reasons account for this difference:
  - Slope of the yield curve;
    - ❖ $C_0^A = B_0[1 + \frac{1}{2}(f_5 - f_1)]$
  - Mismatch between the duration of the crediting rates and the interval between
    - ❖ Assume that a plan credits at one-year zero-coupon bond rate;
    - ❖ $C_0 = B_0 \frac{(1+f_1+f_2+f_3+f_4)}{(1+f_1+f_2+f_3+f_4)} = B_0$

Numerical Example – Effective Duration

- $C_0^A = B_0[1 + \frac{1}{2}(f_5 - f_1)]$

- In the case of a parallel shift in yield curve, present-value cost of Liability A is invariant.

- More realistically, an event that results in large increases in f_1 and f_2 will likely generate smaller increases in f_4 and f_5. In other words, shift in yield curve is usually not parallel.

Comprehensive Analysis

- Cost and effective duration of different cash-balance liabilities are evaluated on the yield curve as of November 15th, 1999.

- For each underlying crediting rates, cost and effective duration are calculated under these scenarios:
  - (1) with and without IRS recommended margin.
  - (2) three time-to-exits (10 years, 20 years and 30 years)
  - (3) random and nonrandom interest rate models

Comprehensive Analysis – No Margin

- Stochastic Interest Rate

no margins	10yr maturity		20yr maturity		30yr maturity	
random model	cost	effdur	cost	effdur	cost	effdur
3mo discount	0.962	0.599	0.930	1.046	0.905	1.321
6mo discount	0.959	0.754	0.923	1.315	0.894	1.664
12mo discount	0.955	1.061	0.907	1.847	0.871	2.342
1yr yield	0.990	0.247	0.979	0.428	0.971	0.537
2yr yield	0.998	0.399	0.983	0.678	0.973	0.858
3yr yield	1.004	0.546	0.985	0.921	0.973	1.169
5yr yield	1.013	0.822	0.985	1.376	0.972	1.758
7yr yield	1.018	1.072	0.982	1.791	0.968	2.299
10yr yield	1.020	1.395	0.973	2.338	0.958	3.025
20yr yield	1.003	2.113	0.939	3.650	0.923	4.823
30yr yield	0.989	2.542	0.920	4.491	0.904	6.003

- Deterministic Interest Rate

no margins	10yr maturity		20yr maturity		30yr maturity	
certainty model	cost	effdur	cost	effdur	cost	effdur
3mo discount	0.963	0.596	0.935	1.038	0.913	1.305
6mo discount	0.961	0.752	0.928	1.307	0.902	1.647
12mo discount	0.956	1.059	0.913	1.838	0.880	2.325
1yr yield	0.991	0.246	0.981	0.423	0.974	0.530
2yr yield	0.998	0.398	0.984	0.674	0.974	0.851
3yr yield	1.004	0.545	0.985	0.917	0.974	1.163
5yr yield	1.012	0.821	0.984	1.372	0.972	1.752
7yr yield	1.017	1.070	0.981	1.787	0.967	2.295
10yr yield	1.019	1.394	0.971	2.335	0.957	3.021
20yr yield	1.002	2.115	0.937	3.651	0.922	4.824
30yr yield	0.988	2.546	0.918	4.495	0.903	6.007

Comprehensive Analysis – with Margin

- Stochastic Interest Rate

IRS margins	10yr maturity		20yr maturity		30yr maturity	
random model	cost	effdur	cost	effdur	cost	effdur
3mo discount	1.133	0.727	1.290	1.266	1.479	1.606
6mo discount	1.104	0.863	1.222	1.503	1.362	1.907
12mo discount	1.099	1.166	1.202	2.030	1.329	2.582
1yr yield	1.087	0.323	1.181	0.557	1.286	0.704
2yr yield	1.045	0.437	1.079	0.742	1.120	0.941
3yr yield	1.052	0.584	1.082	0.985	1.120	1.252
5yr yield	1.037	0.841	1.033	1.408	1.043	1.799
7yr yield	1.042	1.090	1.029	1.822	1.038	2.340
10yr yield	1.020	1.395	0.973	2.338	0.958	3.025
20yr yield	1.003	2.113	0.939	3.650	0.923	4.823
30yr yield	0.989	2.542	0.920	4.491	0.904	6.003

- Deterministic Interest Rate

IRS margins	10yr maturity		20yr maturity		30yr maturity	
certainty model	cost	effdur	cost	effdur	cost	effdur
3mo discount	1.135	0.726	1.297	1.259	1.493	1.592
6mo discount	1.106	0.861	1.228	1.495	1.375	1.892
12mo discount	1.101	1.165	1.209	2.023	1.343	2.567
1yr yield	1.088	0.322	1.183	0.553	1.290	0.697
2yr yield	1.046	0.436	1.080	0.738	1.122	0.934
3yr yield	1.052	0.582	1.082	0.980	1.122	1.246
5yr yield	1.036	0.839	1.032	1.404	1.043	1.793
7yr yield	1.041	1.088	1.028	1.818	1.038	2.335
10yr yield	1.019	1.394	0.971	2.335	0.957	3.021
20yr yield	1.002	2.115	0.937	3.651	0.922	4.824
30yr yield	0.988	2.546	0.918	4.495	0.903	6.007

Comprehensive Analysis – Impact of IRS Margin

- Deterministic Interest Rate
  - ❑ No IRS Margin

no margins	10yr maturity		20yr maturity		30yr maturity	
certainty model	cost	effdur	cost	effdur	cost	effdur
3mo discount	0.963	0.596	0.935	1.038	0.913	1.305
6mo discount	0.961	0.752	0.928	1.307	0.902	1.647
12mo discount	0.956	1.059	0.913	1.838	0.880	2.325
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- Deterministic Interest Rate
  - ❑ With IRS Margin

IRS margins	10yr maturity		20yr maturity		30yr maturity	
certainty model	cost	effdur	cost	effdur	cost	effdur
3mo discount	1.135	0.726	1.297	1.259	1.493	1.592
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Comprehensive Analysis – Effective Duration

There are time variables in the specification of a cash-balance plan that might look like effective duration. But they are not. By examining why different likely candidates are not the correct duration, we can develop intuition for how the effective duration is determined.

- Effective duration is **NOT** the **time-to-maturity of the cash-balance liability**. Even though there is no intermittent cash flows, the final pay-off at maturity of a cash-balance is indexed to interest rates.

- Effective duration is **NOT** the **duration of the underlying asset.** In most cases, effective duration is smaller than the duration of the underlying asset due to resets.

- Effective duration is **NOT** the **time between crediting rates reset**. Recall that there are two rates associated with a cash-balance plan i.e. the crediting rate and discount rate.

Approximation Formula - Cost

- For crediting rates with shorter terms*, their compounding can be approximated using: $(1+x)*(1+y) \approx 1+x+y$ under reasonable rate and yield curve slope assumptions.

- Assume a plan crediting at 3-year zero-coupon bond yield has a maturity T much larger than 3 years. Its cost can be approximated using the following formula.

$$
\begin{aligned}
C_0 &= B_0 \frac{(1+r_1^c)(1+r_2^c)(1+r_3^c)...(1+r_T^c)}{(1+r_1)(1+r_2)(1+r_3)...(1+r_T)} \\
&\approx B_0 \frac{(1+f_0^1/3)(1+2f_0^2/3)(1+f_0^3)...(1+f_0^T)(1+2f_0^{T+1}/3)(1+f_0^{T+2}/3)}{(1+r_1)(1+r_2)(1+r_3)...(1+r_T)} \\
&= B_0 \frac{(1+2f_0^{T+1}/3)(1+f_0^T/3)}{(1+2f_0^1/3)(1+f_0^2/3)} \\
&\approx B_0(1+f_0^T-f_0^1),
\end{aligned}
$$

- More generally, this formula can be rewritten for a plan that credits at *M-month* discount yield, has a Maturity $T > M$ and IRS margin of π

$$C_0 = B_0\left(1+\frac{M-1}{2}(f_0^T-f_0^1)\right)(1+\pi)^T$$

Approximation Formula – Effective Duration

- In order to arrive at the effective duration, an interest rate model must be selected first. This paper uses simple Vasicek model, which means the interest rate is mean-reverting.

- The effective duration is defined as the maturity of a zero-coupon bond with the same cost and same sensitivity to an interest rate shock.

- Based on Vasicek's bond pricing formula, the impact of a shock δ in the instantaneous interest rate on the zero-coupon bond price D_0^t

$$\frac{dD_0^t}{d\delta} = -\frac{1-e^{-kt}}{k} D_0^t$$

- Solving for t obtains the expression for the effective duration of a zero-coupon bond in terms of its sensitivity to shock. By definition of effective duration, this expression must hold for all claims with duration t and cost D_0^t/C_0

$$\begin{aligned} t &= -\frac{1}{k}\log\left(1 + k\frac{1}{D_0^t}\frac{dD_0^t}{d\delta}\right) \\ &= -\frac{1}{k}\log\left(1 + k\frac{1}{C_0}\frac{dC_0}{d\delta}\right) \\ &= -\frac{1}{k}\log\left(1 + k\frac{d\log(C_0)}{d\delta}\right) \end{aligned}$$

Approximation Formula – Effective Duration

$$t = -\frac{1}{k}\log\left(1 + k\frac{d\log(C_0)}{d\delta}\right)$$

The formula above describes the effective duration for any liability whose cost is C_0. In the limit as k approaches 0, this formula becomes the same as what would be implied from traditional Macauley duration based on parallel shift of yield curves.

One simple way to calculate the derivative in the formula is to calculate it numerically by considering direct shocks to the initial yield curve.

$$\frac{d\log(dC_0)}{d\delta} \approx \frac{C_{0,up} - C_{0,down}}{2\delta}$$

where $C_{0,up}$ is the present value cost of obligation after an increase in interest rate and $C_{0,down}$ is the present value cost of obligation after a decrease in interest rate

Conclusions

- The cost of funding cash-balance plans depends on the current slope of the term structure, any margin associated with various crediting assets sanctioned by IRS, time to maturity and volatility of interest rates.

- The effective duration of cash-balance plans are significantly lower than time-to-maturity due to incomplete cancelation in rate movements for crediting and discounting.

Implications

- IRS margins are expensive. Alas, the guideline encourages companies to use longer term rates (e.g. 10 yr/30yr) as crediting rates.

- If the company bases its discount rates on treasury rates, they can hedge their interest rate risks by matching effective duration with long position in Treasury STRIPS (preferably with coupon STRIPS, due to their shorter maturity and thus lower convexity).

Future Study

- Expand this analysis to include employer or employee's option to cash out the pension liability by terminating employment
- Impact of minimum interest crediting rule on the cost and effective duration
- What are the key rate durations of a cash-balance plan?
- How would different discount rates (e.g. Citigroup Pension Plan Rate) impact costs, effective duration and hedging strategies?

Hedging Strategies

Since most cash-balance pension plans credit at treasury rate and discount based on corporate bond rates, they face the risk of credit spread narrowing. There are two potential strategies:

- Sell credit default swaps on a basket of high-quality corporate bonds (i.e. CDX).
- Hold high-quality corporate bonds and sell treasury futures.

Strategy 1 – Sell CDX

- By selling CDXs, the sponsor earns a premium and captures the interest rate spread.

- When the credit spread dips low enough, the sponsor can buy the CDXs at a lower rate and earn the difference between CDX premium earned (higher) and CDX premium paid (lower).

- The difference earned can offset the increase in liability from tightening between the crediting rate and discounting rate.

- While this strategy can hedge the credit spread risks, it comes with its own risks such as credit risk and liquidity risks.

Strategy 2 – Sell Treasury Futures

- For sponsors with many investment-grade bonds in their portfolios, they can hedge against the narrowing credit spread by selling Treasury futures.
- When credit spread narrows, the sponsor benefits from its long position in corporate bond as corporate bond rate decreases relatively and from its short position in Treasury as treasury rate increases relatively.
- This strategy is especially useful for cash-balance plans with large payouts resembling those in a traditional defined-benefit plans.*

*This usually happens when the cash-balance plan has large legacy/retiree liabilities